Many Body Problems with "Spin"-Related Contact Interactions

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Abstract

We study quantum mechanical systems with "spin"-related contact interactions in one dimension. The boundary conditions describing the contact interactions are dependent on the spin states of the particles. In particular we investigate the integrability of N-body systems with δ-interactions and point spin couplings. Bethe ansatz solutions, bound states and scattering matrices are explicitly given. The cases of generalized separated boundary condition and some Hamiltonian operators corresponding to special spin related boundary conditions are also discussed.

Quantum mechanical solvable models describing a particle moving in a local singular potential concentrated at one or a discrete number of points have been extensively discussed in the literature, see e.g. [6, 8, 14] and references therein. One dimensional problems with contact interactions at, say, the origin ($x = 0$) can be characterized by the boundary conditions imposed on the (scalar) wave function φ at $x = 0$. The history of this problem is well described in [6, 8]. It was suggested to divide these conditions into two disjoint families: separated and nonseparated boundary conditions, corresponding to the cases where the perturbed operator is equal to the orthogonal sum of two self–adjoint operators in $L_2(-\infty, 0]$ and $L_2[0, \infty)$ and when this representation is impossible, respectively. Classification of one dimensional point interactions in terms of singular perturbations is given in [19]. In the present paper we are interested in model few-body problems with pairwise interactions given by such potentials. The first model of this type with the pairwise interactions determined by delta functions was suggested and investigated by J.B. McGuire and C.A. Hurst [24, 25, 26, 27, 28]. The eigenfunctions for the system of identical particles interacting via delta potentials are given by Bethe Ansatz. Intensive studies of this model applied to statistical mechanics (particles having boson or fermion statistics) by C.N.Yang and his collaborators lead to the famous Yang-Baxter equation [15, 30, 31]. It has been shown in [9, 20, 18] that N-particle systems with three-body interactions do not have eigenfunctions given by Bethe Ansatz. In [3] the integrability of one dimensional systems of N identical particles with general contact interactions described by the boundary conditions imposed on the wave function was investigated. It was shown that the N-particle system satisfies a Yang-Baxter relation not only in the δ-interaction case, but also for two other one parameter (sub)families, one with nonseparated boundary conditions and another with separated boundary conditions. This fact is not surprising, since the Yang-Baxter equation has been derived for particles with boson or fermion statistics. Suppose that the system of N particles satisfies one of these statistics. Then the eigenfunction equation can be reduced to an equation in the sector $x_1 \leq x_2 \leq \ldots \leq x_N$, since the value of the total wave function in the whole space $\mathbf{R}^N$ can be reconstructed using symmetry properties of this wave function. The boundary conditions on the total wave function are transferred into certain conditions at the boundaries of the sector for the reduced wave function. In fact all three families of

boundary conditions obtained in [3] correspond to one reduced problem. Hence as far as particles with statistics are concerned the only difference between the three families is due to the symmetry properties of the wave function, i.e. the rule how the total wave function can be reconstructed from the reduced one. Considering particles without any statistics the eigenfunctions corresponding to the boundary conditions from the three one parameter families the eigenfunctions can be calculated using Bethe Ansatz. In [5] it is shown that not only the models satisfying the Yang-Baxter equation have eigenfunctions of the type of those constructed following Bethe Ansatz. This is possible, since to derive Yang-Baxter equation from Bethe Ansatz one has to use symmetry properties of the wave function determined by the statistics. The family of such model operators is described by two real parameters. One of these parameters is redundant in the sense that the operators corresponding to different values of this parameter are unitary equivalent. It is shown in [5] that the redundant parameter can be interpreted as the amplitude of a singular gauge field. Note that this parameter can play an important role for nonstationary problems. A similar problem has been studied in [12] but it was wrongly concluded there that the family of such models having eigenfunctions given by Bethe Ansatz coincides with the family of models satisfying the Yang-Baxter equation. This point has been already clarified in [5].

The family of point interactions for the one dimensional Schrödinger operator $-\frac{d^2}{dx^2}$ can be described by unitary 2×2 matrices via von Neumann formulas for self–adjoint extensions of symmetric operators, since the second derivative operator restricted to the domain $C_0^\infty(\mathbf{R} \setminus \{0\})$ has deficiency indices $(2, 2)$. In what follows we are going to consider only the self–adjoint nonseparated extensions that cannot be presented as an orthogonal sum of two self–adjoint operators acting in $L_2(-\infty, 0]$ and $L_2[0, \infty)$. The boundary conditions describing the self-adjoint extensions have the following form

$$\begin{pmatrix} \varphi \\ \varphi' \end{pmatrix}_{0+} = e^{i\theta} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \varphi \\ \varphi' \end{pmatrix}_{0-}, \tag{1}$$

where

$$ad - bc = 1, \quad \theta, a, b, c, d \in \mathbb{R}. \tag{2}$$

$\varphi(x)$ is the scalar wave function of two particles with spin 0 and relative coordinate x. (1) also describes two particles with spin s but without any spin coupling between

the particles when they meet (i.e. for $x = 0$), in this case φ represents any one of the components of the wave function. The values $\theta = b = 0$, $a = d = 1$ in (1) correspond to the case of a positive (resp. negative) δ-function potential for $c > 0$ (resp. $c < 0$). For general a, b, c and d, the properties of the corresponding Hamiltonian systems have been studied in detail, see e.g. [1, 2, 4, 11, 19, 29].

For a particle with spin s, the wave function has $n = 2s + 1$ components. Therefore two particles with contact interactions have a general boundary condition described in the center of mass coordinate system by:

$$\begin{pmatrix} \psi \\ \psi' \end{pmatrix}_{0+} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \psi \\ \psi' \end{pmatrix}_{0-}, \tag{3}$$

where ψ and ψ' are n^2-dimensional column vectors, A, B, C and D are $n^2 \times n^2$ matrices. The boundary condition (3) can include not only the usual contact interaction between the particles, but also a spin coupling of the two particles if the matrices A, B, C, D are not diagonal. These conditions are similar to those appeared in [7, 21] during the investigation of finite rank singular perturbations of differential operators.

The matrices A, B, C, and D are subject to restrictions due to the required symmetry condition of the Schrödinger operator. In fact we should have, for any $u, v \in C^\infty(\mathbb{R}\setminus\{0\})$,

$$
\begin{aligned}
&< -\frac{d^2}{dx^2}u, v >_{L_2(\mathbb{R}, \mathbb{C}^n)} - < u, -\frac{d^2}{dx^2}v >_{L_2(\mathbb{R}, \mathbb{C}^n)} \\
&= < u'(0^+), v(0^+) >_{\mathbb{C}^n} - < u(0^+), v'(0^+) >_{\mathbb{C}^n} \\
&\quad - < u'(0^-), v(0^-) >_{\mathbb{C}^n} + < u(0^-), v'(0^-) >_{\mathbb{C}^n} = 0.
\end{aligned}
\tag{4}
$$

¿From (3) and (4) we get the following conditions:

$$A^\dagger D - C^\dagger B = 1, \quad B^\dagger D = D^\dagger B, \quad A^\dagger C = C^\dagger A, \tag{5}$$

where $\dagger$ stands for the conjugate and transpose. Obviously (1) is the special case of (3) at $s = 0$.

In the following we study quantum systems with contact interactions described by the boundary condition (3), in particular, N-body systems with δ-interactions. We first consider two spin-s particles with δ-interactions. The Hamiltonian is then of the form

$$H = (-\frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2})\mathbf{I}_2 + 2h\delta(x_1 - x_2), \tag{6}$$

4

where $\mathbf{I}_2$ is the $n^2 \times n^2$ identity matrix, h is an $n^2 \times n^2$ Hermitian matrix. If the matrix h is proportional to the unit matrix $\mathbf{I}_2$, then H is reduced to the usual two-particle Hamiltonian with contact interactions but no spin coupling.

Let e_α, $\alpha = 1, ..., n$, be the basis (column) vector with the α-th component as 1 and the rest components 0. The wave function of the system (6) is of the form

$$\psi = \sum_{\alpha,\beta=1}^{n} \phi_{\alpha\beta}(x_1, x_2)e_\alpha \otimes e_\beta. \tag{7}$$

In the center of mass coordinate system, $X = (x_1 + x_2)/2$, $x = x_1 - x_2$, the operator (6) has the form

$$H = -\left(\frac{1}{2}\frac{\partial^2}{\partial X^2} + 2\frac{\partial^2}{\partial x^2}\right)\mathbf{I}_2 + 2h\delta(x). \tag{8}$$

The functions $\phi = \phi(x, X)$ from the domain of this operator satisfy the following boundary condition at $x = 0$,

$$\phi'_{\alpha\beta}(0^+, X) - \phi'_{\alpha\beta}(0^-, X) = \sum_{\alpha,\beta=1}^{n} h_{\gamma\lambda,\alpha\beta}\phi_{\gamma\lambda}(0, X), \quad \phi_{\alpha\beta}(0^+, X) = \phi_{\alpha\beta}(0^-, X), \quad \alpha, \beta = 1, ..., n, \tag{9}$$

where the indices of the matrix h are arranged as $11, 12, ..., 1n; 21, 22, ..., 2n; ...; n1, n2, ..., nn$. (9) is a special case of (3) for $A = D = \mathbf{I}_2$, $B = 0$ and $C = h$. h acts on the basis vector of particles 1 and 2 by $he_\alpha \otimes e_\beta = \sum_{\gamma,\lambda=1}^{n} h_{\alpha\beta,\gamma\lambda}e_\gamma \otimes e_\lambda$.

According to the statistics ψ is symmetric (resp. antisymmetric) under the interchange of the two particles if s is an integer (resp. half integer). Let k_1 and k_2 be the momenta of the two particles. In the region $x_1 < x_2$, in terms of Bethe hypothesis the wave function has the following form

$$\psi = u_{12}e^{i(k_1x_1+k_2x_2)} + u_{21}e^{i(k_2x_1+k_1x_2)}, \tag{10}$$

where u_{12} and u_{21} are $n^2 \times 1$ column matrices. In the region $x_1 > x_2$,

$$\psi = (P^{12}u_{12})e^{i(k_1x_2+k_2x_1)} + (P^{12}u_{21})e^{i(k_2x_2+k_1x_1)}, \tag{11}$$

where according to the symmetry or antisymmetry conditions, $P^{12} = p^{12}$ for bosons and $P^{12} = -p^{12}$ for fermions, p^{12} being the operator on the $n^2 \times 1$ column that interchanges the spins of the two particles. Substituting (10) and (11) into the boundary conditions

(9), we get

$$\begin{cases} u_{12} + u_{21} = P^{12}(u_{12} + u_{21}), \\ ik_{12}(u_{21} - u_{12}) = hP^{12}(u_{12} + u_{21}) + ik_{12}P^{12}(u_{12} - u_{21}), \end{cases} \tag{12}$$

where $k_{12} = (k_1 - k_2)/2$. Eliminating the term $P^{12}u_{12}$ from (12) we obtain the relation

$$u_{21} = Y_{21}^{12} u_{12} , \tag{13}$$

where

$$Y_{21}^{12} = [2ik_{12} - h]^{-1}[2ik_{12}P^{12} + h]. \tag{14}$$

For a system of N identical particles with δ-interactions, the Hamiltonian is given by

$$H = -\sum_{i=1}^{N} \frac{\partial^2}{\partial x_i^2} \mathbf{I}_N + \sum_{i<j}^{N} h_{ij}\delta(x_i - x_j), \tag{15}$$

where $\mathbf{I}_N$ is the $n^N \times n^N$ identity matrix, h_{ij} is an operator acting on the i-th and j-th bases as h and the rest as identity, e.g., $h_{12} = h \otimes \mathbf{1}_3 \otimes ...\mathbf{1}_N$, with $\mathbf{1}_i$ the $n \times n$ identity matrix acting on the i-th basis. The wave function in a given region, say $x_1 < x_2 < ... < x_N$, is of the form

$$\begin{aligned} \Psi &= \sum_{\alpha_1,...,\alpha_N=1}^{n} \phi_{\alpha_1,...,\alpha_N}(x_1,...,x_N)e_{\alpha_1} \otimes ... \otimes e_{\alpha_N} \\ &= u_{12...N}e^{i(k_1x_1+k_2x_2+...+k_Nx_N)} + u_{21...N}e^{i(k_2x_1+k_1x_2+...+k_Nx_N)} + (N! - 2)\,\text{other terms}, \end{aligned} \tag{16}$$

where k_j, $j = 1,...,N$, are the momentum of the j-th particle. u are $n^N \times 1$ matrices. The wave functions in the other regions are determined from (16) by the requirement of symmetry (for bosons) or antisymmetry (for fermions). Along any plane $x_i = x_{i+1}$, $i \in 1, 2, ..., N-1$, we have

$$u_{\alpha_1\alpha_2...\alpha_j\alpha_{j+1}...\alpha_N} = Y_{\alpha_{j+1}\alpha_j}^{jj+1} u_{\alpha_1\alpha_2...\alpha_{j+1}\alpha_j...\alpha_N}, \tag{17}$$

where

$$Y_{\alpha_{j+1}\alpha_j}^{jj+1} = [2ik_{\alpha_j\alpha_{j+1}} - h_{jj+1}]^{-1}[2jk_{\alpha_j\alpha_{j+1}}P^{jj+1} + h_{jj+1}]. \tag{18}$$

Here $k_{\alpha_j\alpha_{j+1}} = (k_{\alpha_j} - k_{\alpha_{j+1}})/2$ play the role of momenta and $P^{jj+1} = p^{jj+1}$ for bosons and $P^{jj+1} = -p^{jj+1}$ for fermions, where p^{jj+1} is the operator on the $n^N \times 1$ column u that interchanges the spins of particles j and $j+1$.

For consistency Y must satisfy the Yang-Baxter equation with spectral parameter [30, 31],

$$Y_{ij}^{m,m+1}Y_{kj}^{m+1,m+2}Y_{ki}^{m,m+1} = Y_{ki}^{m+1,m+2}Y_{kj}^{m,m+1}Y_{ij}^{m+1,m+2}, \quad (19)$$

or

$$Y_{ij}^{mr}Y_{kj}^{rs}Y_{ki}^{mr} = Y_{ki}^{rs}Y_{kj}^{mr}Y_{ij}^{rs}$$

if m, r, s are all unequal, and

$$Y_{ij}^{mr}Y_{ji}^{mr} = 1, \qquad Y_{ij}^{mr}Y_{kl}^{sq} = Y_{kl}^{sq}Y_{ij}^{mr} \quad (20)$$

if m, r, s, q are all unequal. By a straightforward calculation it can be shown that the operator Y given by (18) satisfies all the Yang-Baxter relations if

$$[h_{ij}, P^{ij}] = 0. \quad (21)$$

Therefore if the Hamiltonian operators for the spin coupling commute with the spin permutation operator, the N-body quantum system (15) can be exactly solved. The wave function is then given by (16) and (17) with the energy $E = \sum_{i=1}^{N} k_i^2$.

For the case of spin-$\frac{1}{2}$, a Hermitian matrix satisfying (21) is generally of the form

$$h^{\frac{1}{2}} = \begin{pmatrix} a & e_1 & e_1 & c \\ e_1^* & f & g & e_2 \\ e_1^* & g & f & e_2 \\ c^* & e_2^* & e_2^* & b \end{pmatrix}, \quad (22)$$

where $a, b, c, f, e_1, e_2 \in \mathbb{C}$, $g \in \mathbb{R}$. We recall that for a complex vector space V, a matrix R taking values in $End_c(V \otimes V)$ is called a solution of the Yang-Baxter equation without spectral parameters, if it satisfies

$$\mathcal{R}_{12}\mathcal{R}_{13}\mathcal{R}_{23} = \mathcal{R}_{23}\mathcal{R}_{13}\mathcal{R}_{12}, \quad (23)$$

where $\mathcal{R}_{ij}$ denotes the matrix on the complex vector space $V \otimes V \otimes V$, acting as R on the i-th and the j-th components and as identity on the other components. When V is a two dimensional complex space, the solutions of (23) include the ones such as R_q which gives rise to the quantum algebra $SU_q(2)$ and the integrable Heisenberg spin-$\frac{1}{2}$ chain models such as the XXZ model (R corresponds to the spin coupling operator between the nearest neighbor spins in Heisenberg spin chain models)[10, 17, 22, 23]. Nevertheless

in general $h^{\frac{1}{2}}$ does not satisfy the Yang-Baxter equation without spectral parameters: $h^{\frac{1}{2}}_{12}h^{\frac{1}{2}}_{13}h^{\frac{1}{2}}_{23} \neq h^{\frac{1}{2}}_{23}h^{\frac{1}{2}}_{13}h^{\frac{1}{2}}_{12}$. But (22) includes the Yang-Baxter solutions, such as R_q, that gives integrable spin chain models (for an extensive investigation of the Yang-Baxter solutions see [13, 16]). Therefore for an N-body system to be integrable, the spin coupling in the contact interaction (15) is allowed to be more general than the spin coupling in a Heisenberg spin chain model with nearest neighbors interactions.

We now investigate the problem of bound states. For $N = 2$, from (12) the bound states have the form,

$$\psi^2_\alpha = u_\alpha e^{\frac{c+a\Lambda_\alpha}{2}|x_2-x_1|}, \qquad \alpha = 1, ..., n^2, \tag{24}$$

where u_α is the common α-th eigenvector of h and P^{12}, with eigenvalue Λ_α, s.t. $hu_\alpha = \Lambda_\alpha u_\alpha$ and $c + a\Lambda_\alpha < 0$, $P^{12}u_\alpha = u_\alpha$. The eigenvalue of the Hamitonian H corresponding to the bound state (24) is $-(c + a\Lambda_\alpha)^2/2$. We remark that, whereas for the case of the boundary condition (1), for a δ interaction one has a unique bound state, here we have n^2 bound states.

By generalization we get the bound state for the N-particle system,

$$\psi^N_\alpha = v_\alpha e^{-\frac{c+a\Lambda_\alpha}{2}\sum_{i>j}|x_i-x_j|}, \qquad \alpha = 1, ..., n^2, \tag{25}$$

where v_α is the wave function of the spin part.

It can be checked that ψ^N_α satisfy the boundary condition (9) at $x_i = x_j$ for any $i \neq j \in 1, ..., N$. The spin wave function v here satisfies $P^{ij}v_\alpha = v_\alpha$ and $h_{ij}v_\alpha = \Lambda_\alpha v_\alpha$, for any $i \neq j$.

It is worth mentioning that ψ^N_α is of the form (16) in each of the above regions. For instance comparing ψ^N_α with (16) in the region $x_1 < x_2... < x_N$ we get

$$k_1 = -i\frac{c + a\Lambda_\alpha}{2}(N - 1), \; k_2 = k_1 + ic, \; k_3 = k_2 + ic, ..., k_N = -k_1, \tag{26}$$

for $\alpha = 1, ..., n^2$. The energy of the bound state ψ^N_α is

$$E_\alpha = -\frac{(c + a\Lambda_\alpha)^2}{12}N(N^2 - 1). \tag{27}$$

Now we pass to the scattering matrix. For real $k_1 < k_2 < ...k_N$, in each coordinate region such as $x_1 < x_2 < ...x_N$, the following term in (16) describes an outgoing wave

$$\psi_{out} = u_{12...N}e^{i(k_1x_1+...+k_Nx_N)}. \tag{28}$$

An incoming wave with the same exponential as (28) is given by

$$\psi_{in} = [P^{1N}P^{2(N-1)}...]u_{N(N-1)...1}e^{i(k_N x_N + ... + k_1 x_1)} \tag{29}$$

in the region $x_N < x_{N-1} < ... < x_1$. From (17) the scattering matrix S defined by $\psi_{out} = S\psi_{in}$ is given by

$$S = [X_{21}X_{31}...X_{N1}][X_{32}X_{42}...X_{N2}]...[X_{N(N-1)}], \tag{30}$$

where $X_{ij} = Y_{ij}^{ij}P^{ij}$.

The scattering matrix S is unitary and symmetric due to the time reversal invariance of the interactions. $< s_1' s_2'...s_N'|S|s_1 s_2...s_N >$ stands for the S matrix element of the process from the state $(k_1 s_1, k_2 s_2, ..., k_N s_N)$ to the state $(k_1 s_1', k_2 s_2', ..., k_N s_N')$.

The scattering of clusters (bound states) can be discussed in a similar way as in [15]. For instance for the scattering of a bound state of two particles $(x_1 < x_2)$ on a bound state of three particles $(x_3 < x_4 < x_5)$, the scattering matrix is $S = [X_{32}X_{42}X_{52}][X_{31}X_{41}X_{51}]$.

The integrability of many particles system with contact spin coupling interactions governed by separated boundary conditions can also be studied. Instead of (3) we need to deal with the case

$$\phi'(0_+) = G^+\phi(0_+), \quad \phi'(0_-) = G^-\phi(0_-), \tag{31}$$

where $G^\pm$ are Hermitian matrices. For $G^+ = G^- \equiv G$, $G^\dagger = G$, there is a Bethe Ansatz solution to (16) with $Y_{l_{i+1}l_i}^{ii+1}$ in (17) given by

$$Y_{l_{i+1}l_i}^{ii+1} = \frac{ik_{l_i l_{i+1}} + G}{ik_{l_i l_{i+1}} - G}. \tag{32}$$

Let Γ be the set of n^2 eigenvalues of G. For any $\lambda_\alpha \in \Gamma$ such that $\lambda_\alpha < 0$, there are $2^{N(N-1)/2}$ bound states for the N-particle system,

$$\psi_{\alpha\underline{\epsilon}}^N = v_{\alpha\underline{\epsilon}} \prod_{k>l}(\theta(x_k - x_l) + \epsilon_{kl}\theta(x_l - x_k))e^{\lambda_\alpha \sum_{i>j}|x_i - x_j|}, \tag{33}$$

where $v_{\alpha\underline{\epsilon}}$ is the spin wave function and $\underline{\epsilon} \equiv \{\epsilon_{kl} : k > l\}$; $\epsilon_{kl} = \pm$, labels the $2^{N(N-1)/2}$-fold degeneracy. The spin wave function v here satisfies $P^{ij}v_{\alpha\underline{\epsilon}} = \epsilon_{ij}v_{\alpha\underline{\epsilon}}$ for any $i \neq j$, that is, $p^{ij}v_{\alpha\underline{\epsilon}} = \epsilon_{ij}v_{\alpha\underline{\epsilon}}$ for bosons and $p^{ij}v_{\alpha\underline{\epsilon}} = -\epsilon_{ij}v_{\alpha\underline{\epsilon}}$ for fermions.

Again $\psi_{\alpha\underline{\epsilon}}^N$ is of the form (16) in each of the regions $x_{i_1} < x_{i_2} < ... < x_{i_N}$. For instance comparing $\psi_{\alpha\underline{\epsilon}}^N$ with (16) in the region $x_1 < x_2... < x_N$ we get $k_1 = i\lambda_\alpha(N-1)$, $k_2 = k_1 - 2i\lambda_\alpha$, $k_3 = k_2 - 2i\lambda_\alpha,...,k_N = -k_1$. The energy of the bound state $\psi_{\alpha\underline{\epsilon}}^N$ is

$$E_\alpha = -\frac{\lambda_\alpha^2}{3}N(N^2-1). \tag{34}$$

We have investigated the integrable models of N-body systems with contact spin coupling interactions. Without taking into account the spin coupling, the boundary condition (1) is characterized by four parameters (separated boundary conditions are a special limiting case of these). Obviously the general boundary condition (3) we considered in this article has much more parameters. The classification of the dynamic operators associated with different parameter regions is a big challenge. As we have seen, the case $A = D = \mathbf{I}_2$, $B = 0$, $C = h$ corresponds to a Hamiltonian with δ-interactions of the form (6) (for $N = 2$). It can be further shown that (for $N = 2$) the following boundary condition

$$\begin{pmatrix} \psi \\ \psi' \end{pmatrix}_{0+} = \begin{pmatrix} \mathbf{I} & B \\ 0 & \mathbf{I} \end{pmatrix} \begin{pmatrix} \psi \\ \psi' \end{pmatrix}_{0-}, \tag{35}$$

corresponds to a Hamiltonian H of the form:

$$H = -D_x^2(1 + B\delta) - BD_x\delta',$$

where B is an $n^2 \times n^2$ Hermitian matrix, D_x is defined by $(D_x f)(\varphi) = -f(\frac{d}{dx}\varphi)$, for $f \in C_0^\infty(\mathbb{R}/\{0\})$ and φ a test function with a possible discontinuity at the origin.

The boundary condition

$$\begin{pmatrix} \psi \\ \psi' \end{pmatrix}_{0+} = \begin{pmatrix} \frac{2+iB}{2-iB} & 0 \\ 0 & \frac{2-iB}{2+iB} \end{pmatrix} \begin{pmatrix} \psi \\ \psi' \end{pmatrix}_{0-}, \tag{36}$$

describes the Hamiltonian

$$H = -D_x^2 + iB(2D_x\delta - \delta').$$

We have introduced boundary conditions depending on the spin states of the particles and studied several special cases. A complete investigation of integrable N-body systems and Hamiltonian operators corresponding to the general boundary conditions of the form (3) still remains to be done.

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